In a nutshell: The bracketed secant method

Given a continuous real-valued function f of a real variable defined on the interval $[a_0, b_0]$ where $f(a_0)$ and $f(b_0)$ have opposite signs and neither is zero, the intermediate-value theorem guarantees that there is a root on that interval. This algorithm uses iteration, bracketing, linear interpolation and solving a trivial linear equation to approximate the root. The intermediate-value theorem is used to guarantee the existence of the root.

Parameters:

$\varepsilon_{\text{step}}$	The maximum error in the value of the root cannot exceed this value.
ε_{abs}	The value of the function at the approximation of the root cannot exceed this value.
N	The maximum number of iterations.

1. Let $k \leftarrow 0$.
2. If $k > N$, we have iterated N times, so stop and return signalling a failure to converge.
3. The next approximation to the root will be the root of the linear polynomial that interpolates the points $(a_k, f(a_k))$ and $(b_k, f(b_k))$.

   Let $r_k \leftarrow a_k - f(a_k)\dfrac{b_k - a_k}{f(b_k) - f(a_k)}$.

   a. If $f(r_k) = 0$, we are done, and return m_k.
   b. If $f(r_k)$ and $f(a_k)$ have the same sign,
      i. if $r_k - a_k < \varepsilon_{\text{step}}$ and $|f(r_k)| < \varepsilon_{\text{abs}}$, return r_k,
      ii. otherwise, let $a_{k+1} \leftarrow r_k$ and $b_{k+1} \leftarrow b_k$;
   c. otherwise, $f(r_k)$ and $f(b_k)$ must have the same sign,
      i. if $b_k - r_k < \varepsilon_{\text{step}}$ and $|f(r_k)| < \varepsilon_{\text{abs}}$, return r_k,
      ii. otherwise, let $a_{k+1} \leftarrow a_k$ and $b_{k+1} \leftarrow r_k$.
4. Increment k and return to Step 2.

Convergence

As one end-point tends to become fixed (once the function is either strictly concave up or strictly concave down on the interval $[a_k, b_k]$, Thus, if h is the error, it can be show that the error decreases according to $O(h)$.